International Journal of Theoretical Physics, Vol. 35, No. 8, 1996

Intertwining Relations, Asymmetric Face Model, and Algebraic Bethe Ansatz for $SU_{p,q}(2)$ Invariant Spin Chain

A. Ghose Choudhury[1] and A. Roy Chowdhury[1]

Received August 3, 1995

Intertwining relations for the quantum R-matrix of the $SU_{p,q}(2)$ invariant spin chain are obtained and the corresponding face model is deduced. An important difference is seen to arise due to the asymmetry generated by the parameters p and q, which leads to a asymmetric face model. An algebraic Bethe ansatz is set up and solved with the help of these intertwining vectors.

1. INTRODUCTION

Integrable models of quantum spin chains have played an important role in the development of the quantum inverse scattering transform (Baxter, 1982; Faddeev and Takhtajan, 1979; Wadati and Akutsu, 1988). Besides the usual model of nearest neighbor interaction, other models, such as IRF, SOS, etc., are in vogue and have characteristics of their own (Wadati, 1988; Akutsu, 1987; Deguchi, 1987; Yang and Ge, 1990). It was also demonstrated that the usual spin-chain model can also be transformed into an SOS or IRF model by the use of intertwining relations of the quantum R-matrix (de Vega, 1990, 1992). Here we construct the IRF model pertaining to the quantum R-matrix of the $SU_{p,q}(2)$ invariant spin chain (Das Gupta and Roy Chowdhury, 1993). The Bethe states are subsequently constructed by the intertwining relations. Two parameters p and q give rise to a new feature of our model leading to a different amount of shifting on the lattice.

[1] High Energy Physics Division, Department of Physics, Jadavpur University, Calcutta 700032, India.

0020-7748/96/0800-1699$09.50/0

2. FORMULATION AND DERIVATION OF FACE MODEL

The quantum R-matrix for the $SU_{p,q}(2)$ invariant spin chain can be written as

$$R = \begin{pmatrix} a & 0 & 0 & 0 \\ 0 & b_1 & C & 0 \\ 0 & C & b_2 & 0 \\ 0 & 0 & 0 & a \end{pmatrix} \tag{1}$$

where

$$\begin{aligned} a &= \frac{\lambda}{\mu} q - \frac{\mu}{\lambda} p^{-1}, \qquad C \\ &= q - p^{-1}, \qquad b_1 \\ &= \frac{\lambda}{\mu} - \frac{\mu}{\lambda}, \qquad b_2 \\ &= p^{-1} q \left(\frac{\lambda}{\mu} - \frac{\mu}{\lambda} \right) \end{aligned}$$

We define new variables u and v through $\lambda = e^u$ and $\mu = e^v$ and also set $q = e^\delta$ and $p = e^\gamma$ and consider the transformed R-matrix $\hat{R} = RP$, where P is the permutation matrix. The intertwining relation for the above R-matrix is

$$\hat{R}(u - v)\Phi_1(u) \otimes \Phi_2(V) = h(u - v)[\Phi_3(v) \otimes \Phi_4(u)] \tag{2}$$

A simple analysis of equation (2) shows that possible solutions are

$$\begin{aligned} \Phi_1 &= X^+(u) = \begin{pmatrix} e^{u/2} \\ e^{-u/2} \end{pmatrix}; \qquad \Phi_2 = X^+(v - \gamma) = \begin{pmatrix} e^{(v-r)/2} \\ e^{-(v-r)/2} \end{pmatrix} \\ \Phi_3 &= X^+(v) = \begin{pmatrix} e^{v/2} \\ e^{-v/2} \end{pmatrix}; \qquad \Phi_4 = X^+(u - \gamma) = \begin{pmatrix} e^{(v-\gamma)/2} \\ e^{-(v-\gamma)/2} \end{pmatrix} \end{aligned} \tag{3}$$

and a second set is

$$\begin{aligned} \Phi_1 &= X^-(u) = \begin{pmatrix} e^{-u/2} \\ e^{u/2} \end{pmatrix}; \qquad \Phi_2 = X^-(v - \delta) = \begin{pmatrix} e^{-(v-\delta)/2} \\ e^{(v-\delta)/2} \end{pmatrix} \\ \Phi_3 &= X^-(v); \qquad \Phi_4 = X^-(u - \delta) \end{aligned} \tag{4}$$

One can then easily deduce the following relations for the $X^\pm$ vectors:

$$\hat{R}(u - v)[X^+(u) \otimes X^+(v - r)] = h(u - v)[X^+(v) \otimes X^+(u - \gamma)] \tag{5}$$

with $h = e^{u-v+\delta} - e^{-(u-v+\gamma)}$, and

$$\hat{R}(u - v)[X^-(u) \otimes X^-(v - \delta)] = h(u - v)[X^-(v) \otimes X^-(u - \delta)] \quad (6)$$

$$\begin{aligned}\hat{R}(u - v)[X^+(u) \otimes X^-(v + \alpha)] &= (e^{\delta} - e^{-\gamma})[X^+(v) \otimes X^-(v + \alpha)] \\ &+ e^{(\delta-\gamma)/2}(e^{u-v} - e^{-(u-v)})[X^-(v + \alpha - \gamma) \otimes X^+(u - \delta)] \end{aligned} \quad (7)$$

$$\begin{aligned}\hat{R}(u - v)[X^-(u) \otimes X^+(v + \alpha)] &= (e^{\delta} - e^{-\gamma})[X^-(v) \otimes X^+(u + \alpha)] \\ &+ (e^{u-v} - e^{-(u-v)})e^{(\delta-\gamma)/2}[X^+(v + \alpha - \delta) \otimes X^-(u + \gamma)] \end{aligned} \quad (8)$$

Equations (5)–(8) will be repeatedly used in the following.

We now deduce the statistical weight factors for the corresponding face model by using the above equations. For that we choose the unspecified constant α as follows:

$$\alpha = (k + l)r + (k + m)\delta - s \qquad \text{for } X^+\text{-type vectors}$$

$$\alpha = -(k + l)r - (k + m)\delta + t \qquad \text{for } X^-\text{-type vectors}$$

where k, l, and m run over integers. With this choice we can designate the vectors X, X', Y, and Y' as follows:

$$X = X^+(\theta + \alpha) = X_{l+k}(\theta)$$

$$X' = X^+(\theta + \alpha - \gamma) = X_{l+k-1}(\theta)$$

$$Y = X^-(\theta + \alpha) = Y_{l+k}(\theta)$$

$$Y' = X^-(\theta + \alpha + r) = Y_{l+k-1}(\theta)$$

Thus equations (5)–(8) can be written as

$$\begin{aligned}\hat{R}(u - v)X_l(u) \otimes X_{l-1}(v) &= h(u - v)X_l(v) \otimes X_{l-1}(u) \\ \hat{R}(u - v)Y_l(u) \otimes Y_{l+1}(v) &= h(u - v)Y_l(v) \otimes Y_{l+1}(u) \end{aligned} \quad (9)$$

$$\begin{aligned}\hat{R}(u - v)X_l(u) \otimes Y_p(v) &= (e^{\delta} - e^{-\gamma})X_l(v) \otimes Y_p(u) \\ &+ (e^{u-v} - e^{-u+v})e^{(\delta-\gamma)/2}Y_{p+1}(v) \otimes X_{l+1}(u)\end{aligned}$$

$$\begin{aligned}\hat{R}(u - v)Y_p(u) \otimes X_l(u) &= (e^{\delta} - e^{-\gamma})Y_p(v) \otimes X_l(u) \\ &+ (e^{u-v} - e^{-u+v})e^{(\delta-\gamma)/2}X_{l-1}(v)Y_{p-1}(u)\end{aligned}$$

We now regroup the two classes of vectors X, Y into a single set by defining

$$X^{l,l-1}(u) = X_l(u); \qquad X^{l,l+1}(v) = Y_l(v)$$

Thus the above relations can be written as

$$\hat{R}(u - v)X^{l,l-1}(u) \otimes X^{l-1,l-2}(v) = h(u - v)X^{l,l-1}(v) \otimes X^{l-1,l-2}(u)$$

$$\hat{R}(u - v)X^{l,l+1}(u) \otimes X^{l+1,l+2}(v) = h(u - v)X^{l,l+1}(v) \otimes X^{l+1,l+2}(u)$$

$$\begin{aligned}\hat{R}(u - v)&X^{l,l-1}(u) \otimes X^{p,p+1}(v)\\ &= (e^{\delta} - e^{-\gamma})X^{l,l-1}(v) \otimes X^{p,p+1}(u)\\ &\quad + (e^{u-v} - e^{-u+v})e^{(\delta-\gamma)/2}X^{p+1,p+2}(v) \otimes X^{l+1,l}(u)\end{aligned}$$

$$\begin{aligned}\hat{R}(u - v)&X^{p,p+1}(u) \otimes X^{l,l-1}(v)\\ &= (e^{\delta} - e^{-\gamma})X^{p,p+1}(v) \otimes X^{l,l-1}(u)\\ &\quad + (e^{u-v} - e^{-u+v})e^{(\delta-\gamma)/2}X^{l-1,l-2}(v) \otimes X^{p-1,p}(u)\end{aligned}$$

By a simple readjustment of the indices all these can be combined into a single one,

$$\hat{R}(u - v)X^{l,m}(u) \otimes X^{m,n}(v) = \sum_p W(l, m, n, p\theta)X^{l,p}(v) \otimes X^{p,n}(u) \tag{10}$$

where $\theta = u - v$, along with the condition

$$|l - m| = |m - n| = |l - p| = |p - n| = 1$$

so that we can read off the statistical weight factors for the face model,

$$\begin{aligned}W(l + 1, l, l - 1, l) &= e^{u-v+\delta} - e^{-(u-v+\gamma)}\\ W(l - 1, l, l + 1, l) &= e^{u-v+\delta} - e^{-(u-v+\gamma)}\\ W(l, l - 1, l, l - 1) &= e^{\delta} - e^{-\gamma}\\ W(l, l - 1, l, l + 1) &= (e^{u-v} - e^{-u+v})e^{(\delta-\gamma)/2}\\ W(l, l + 1, l, l + 1) &= e^{\delta} - e^{-\gamma}\\ W(l, l + 1, l, l - 1) &= (e^{u-v} - e^{-u+v})e^{(\delta-\gamma)/2}\end{aligned} \tag{11}$$

3. CONSTRUCTION OF THE VACUUM STATE

We now proceed to the explicit realization of the Bethe states and the evaluation of the corresponding eigenvalues.

The monodromy matrix reads

$$T_{a,b}(\theta) = \sum_{a_1\cdots a_{N-1}} t^{(1)}_{a_1b}(\theta)t^{(2)}_{a_1a_2}(\theta) \cdots t^{(N)}_{aa_{N-1}}(\theta) \tag{12a}$$

where

$$[t_{ab}(\theta)]_{\alpha\gamma} = R^{b\gamma}_{\alpha a}(\theta)$$

A family of gauge-transformed monodromy matrices follows by replacing

$$t^{(k)}_{ab}(\theta)_{\alpha\gamma} \rightarrow [M^{-1}_{k+l}]_{ac} t^{(k)}_{c\alpha}(\theta)[M_{K+l-1}]_{\alpha b} \tag{12b}$$

where M is a 2×2 matrix

$$M = \begin{pmatrix} X_1 & Y_1 \\ X_2 & Y_2 \end{pmatrix} = (X, Y)$$

and

$$M^{-1} = \frac{1}{\Delta}\begin{pmatrix} Y_2 & -Y_1 \\ -X_2 & X_1 \end{pmatrix} = \frac{1}{X \cdot Y}\begin{pmatrix} -\bar{Y} \\ \bar{X} \end{pmatrix}$$

For the monodromy matrix associated with a line in the lattice we find

$$T_{ba}(\theta)_{\alpha|\gamma} \rightarrow [M^{-1}_{l+N}]_{a\alpha} T_{C\alpha}(\theta)_{\alpha|\gamma}[M_l]_{\alpha b} = T^{(l)}_{ab}(\theta)$$

In general we write

$$T^l(\theta) = \begin{pmatrix} A^l(\theta) & B^l(\theta) \\ C^l(\theta) & D^l(\theta) \end{pmatrix} \tag{12c}$$

The first step in setting up the Bethe ansatz is to define a vacuum state, which is obtained from the condition that the "21" element of $\hat{t}(\theta)$ operating on an arbitrary vector $\omega = (\omega_1, \omega_2)^+$ vanishes. That is,

$$\hat{t}_{21}(\theta)\omega = 0 \tag{12d}$$

which is equivalent to

$$\bar{x}'_\eta(\hat{R}X \otimes \omega)_{\sigma\eta} = 0 \tag{13}$$

Comparing with equations (5)–(8), one observes that a possible set of solutions is

$$\begin{aligned} X &= X^+(\theta + \alpha) \\ X' &= X^+(\theta + \alpha - \gamma) \\ \omega &= X^+(\alpha - \gamma) \end{aligned} \tag{14}$$

After the identification of the vacuum, we find the eigenvalue of the state corresponding to the diagonal elements of $\hat{t}(\theta)$. For that we observe that

$$\hat{t}_{11}(\theta)\omega = -h(\theta)\omega' \qquad \text{with} \quad \omega' = X^+(\alpha)$$

or

$$\hat{t}_{11}(\theta)X^+(\alpha - \gamma) = -h(\theta)X^+(\alpha) \tag{15}$$

Similarly,

$$\hat{t}_{22}(\theta)X^+(\alpha - \gamma) = b_1 e^{\delta-\gamma}X^+(\alpha - \gamma - \delta) \tag{16}$$

We now define the arbitrary quantity α occurring in (14) to be

$$\alpha = (k + l)\gamma + (k + m)\delta - s$$

k, l, m integers, for X^+-type vectors, and

$$\alpha = -(k + l)\gamma - (k + m)\delta + t$$

for X^--type vectors, whence we can at once rewrite the solution vectors X^+, X^-, and Y as follows:

$$\begin{aligned} X^+(\theta + \alpha) &= X^+(\theta + (k + l)\gamma + (k + m)\delta - s) \\ &= X_{l+k}(\theta) \end{aligned} \tag{17}$$

$$\begin{aligned} X' &= X^+(\theta + \alpha - \gamma) = X_{l+k-1}(\theta), \qquad X^-(\theta + \alpha) = Y_{l+k}(\theta) \\ Y' &= X^-(\theta + \alpha + \gamma) = Y_{l+k-1}(\theta) \end{aligned} \tag{18}$$

Thus we may say that X', Y' correspond to the $(l + k - 1)$th point of the lattice and X, Y correspond to the $(l + k)$th point. Consequently we get

$$\begin{aligned} \omega &= X^+(\alpha - \gamma) = \omega_k^l \\ \omega' &= X^+(\alpha) = \omega_{k+1}^l \\ \omega'' &= \omega_{k-1}^l \end{aligned} \tag{19}$$

Equations (12), (15), and (16) can be recast as

$$\begin{aligned} \hat{t}_{11}(\theta)\omega_k^l &= h(\theta)\omega_{k+1}^l \\ \hat{t}_{22}(\theta)\omega_k^l &= (e^{\theta} - e^{-\theta})e^{(\delta-\gamma)/2}\omega_{k-1}^l \\ \hat{t}_{21}(\theta)\omega_k^l &= 0 \end{aligned} \tag{20}$$

which permits us to interpret the action of the elements of the monodromy matrix as a shifting on the lattice site.

4. CONSTRUCTION OF THE BETHE STATE

From the above property of the vector ω_k the pseudovacuum can be defined as

$$\Omega_N^l = \omega_1^l \otimes \omega_2^l \otimes \omega_3^l \cdots \otimes \omega_N^l \tag{21}$$

This structure of Ω_N leads at once to

$$\begin{aligned} A(\theta)\Omega_N^l &= h^N(\theta)\Omega_N^{l+1} \\ D^{(\blacksquare)}(\theta)\Omega_N^l &= (e^\theta - e^{-\theta})e^{N/2(\delta-\gamma)}\Omega_N^{l-1} \\ C^{(\blacksquare)}(\theta)\Omega_N^l &= 0 \end{aligned} \tag{22}$$

Next we consider the transformed monodromy and transfer matrix

$$\begin{aligned} T^{(n,l)}(\theta) &= M_n^{-1}(\theta)T(\theta)M_l(\theta); \qquad \tau(\theta) = A_{n,l} + D_{n,l} \\ &= \begin{pmatrix} A_{n,l}(\theta) & B_{n,l}(\theta) \\ C_{n,l}(\theta) & D_{n,l}(\theta) \end{pmatrix} \\ &= \frac{1}{\Delta^{(\theta)}} \begin{bmatrix} \tilde{Y}_n(\theta)T(\theta)X(\theta) & \tilde{Y}_n(\theta)T(\theta)Y(\theta) \\ \tilde{X}_n(\theta)T(\theta)X(\theta) & \tilde{X}_n(\theta)T(\theta)Y(\theta) \end{bmatrix} \end{aligned} \tag{23}$$

The commutation rules of these new monodromy elements can now be deduced by the use of

$$\hat{R}(u-v)Y_l(u) \otimes Y_{l+1}(v) = h(u-v)Y_l(v) \otimes Y_{l+1}(u) \tag{24}$$

and its adjoint

$$\tilde{Y}_{l+1}(v) \otimes \tilde{Y}_l(u)\hat{R}(u-v) = h(u-v)\tilde{Y}_{l+1}(u) \otimes \tilde{Y}_l(v) \tag{25}$$

and using proper projection, which at once yields

$$B_{l+1,k}(u)B_{l,k+1}(v) = B_{l+1,k}(v)B_{l,k+1}(u) \tag{26}$$

$$\begin{aligned} A_{k,n}(v)B_{k-1,n+1}(u) &= \alpha(u-v)B_{k,n+2}(u)A_{k-1,n+1}(v) \\ &\quad - \beta(u-v)\cdot\beta_{k,n+2}(v)A_{k-1,n+1}(u) \end{aligned} \tag{27}$$

$$\begin{aligned} D_{k,l}(u)B_{k-1,l+1}(v) &= \alpha(u-v)B_{k-2,l}(v)D_{k-1,l+1}(v) \\ &\quad - \beta(u-v)B_{k-2,l}(u)D_{k-1,l+1}(v) \end{aligned} \tag{28}$$

where

$$\alpha(u-v) = \frac{h(u-v)\exp((\gamma-\delta)/2)}{e^{u-v} - e^{-u+v}}; \qquad \beta(u-v) = \frac{(e^\delta - e^{-r})\exp((\gamma-\delta)/2)}{e^{u-v} - e^{-u+v}} \tag{29}$$

We now start with the state

$$\Lambda = \beta_{l-1,l+1}(v_1)\beta_{l-2,l+2}(v_2) \cdots \beta_{l-n,l+n}(v_n)\Omega_N^{l+n} \tag{30}$$

and deduce with the help of above communication rules

$$A_{l,l}(u)\Lambda = \prod_{k=1}^{n} \alpha(v_k - u)B_{l,l+2}(v_1)B_{l-1,l+3}(v_2) \cdots B_{l+1-n,l+1+n}(v_n)$$

$$\times A_{l-n,l+n}(u)\Omega_N^{l+n}$$

$$+ \sum_{j=1}^{n} \left\{\beta(u - v_j) \prod_{\substack{k=1\\k\neq j}}^{n} \alpha(v_k - v_j)\right\}B_{l,l+2}(v_1) \cdots B_{l+1-j,l+1+j}(u)$$

$$\times \cdots B_{l+1-n,l+1+n}(v_n)A_{l-n,l+n}(v_j)\Omega_N^{l+n} \tag{31}$$

Now $A_{l-n,l+n}(u)$ can be applied on Ω_N^{l-n} only when $n = N/2$, whence we get, using

$$A^{(p)}(u)\Omega_N^p = h^N(u)\Omega_N^{p+1}$$

$$h(u) = e^{u+\delta} - e^{-(u+\gamma)} \tag{32}$$

that

$$A_{l,l}(u)\psi_l(v_1, v_2, \ldots, v_n)$$

$$= h^N(u) \prod_{k=1}^{n} \alpha(v_k - u)B_{l,l+2}(v_1) \cdots \beta_{l+1-n,l+1+n}(v_n)\Omega_N^{l+n+1}$$

$$+ \sum_{j=1}^{n} \left\{\beta(u - v_j) \prod_{\substack{k=1\\k\neq j}}^{n} \alpha(v_k - v_j)\right\}B_{l,l+2}(v_1) \cdots B_{l+1-j,l+1+j}(u)$$

$$\times \cdots B_{l+1-n,l+1+n}(v_n)h^N(v_j)\Omega_N^{l+1+n} \tag{33}$$

By a similar computation,

$$D_{l,l}(u)\psi_l(v_1 \cdots v_n)$$

$$= h'^N(u) \prod_{k=1}^{n} \alpha(u - v_k)\psi_{l-1}(v_1 \cdots v_n)$$

$$+ \sum_{j=1}^{n} \left\{-\beta(u - v_j) \prod_{\substack{k=1\\k\neq j}}^{n} \alpha(v_j - v_k)\right\}h'^N(v_j)\psi_{l-1}$$

$$\times (v_1 \cdots v_{j-1}, u, v_{j+1} \cdots v_n)$$

$$h'(u) = (e^u - e^{-u})\exp\left(\frac{\delta - \gamma}{2}\right) \tag{34}$$

Now the transfer matrix is given by

$$T(u) = A_{l,l}(u) + D_{l,l}(u) \qquad \text{for all } l$$

From the relations we observe that due to the operation of $A_{l,l}$ or $D_{l,l}$ the index of ψ changes by one and hence they are not eigenvectors. To construct eigenvectors we multiply relations (33) and (34) by $e^{2\pi i l\theta}$, $0 \le \theta \le 1$, and sum over l from $-\infty$ to ∞:

$$\begin{aligned}
&\sum_{l=-\infty}^{\infty} \{A_{l,l}(u) + D_{l,l}(u)\} e^{2\pi i l\theta} \psi_l(v_1 v_2 \cdots v_n) \\
&\quad = \sum_{l=-\infty}^{\infty} \left[h^N(u) \prod_{k=1}^{n} \alpha(v_k - u) e^{2\pi i l\theta} \psi_{l+1}(v_1 \cdots v_n) \right. \\
&\qquad \left. + h'^N(u) \prod_{k=1}^{n} \alpha(u - v_k) e^{2\pi i l\theta} \psi_{l-1}(v_1 \cdots v_n) \right] \\
&\qquad + \sum_{l=-\infty}^{\infty} \sum_{j=1}^{n} e^{2\pi i l\theta} \left[\beta(u - v_j) \prod_{\substack{k=1 \\ k\neq j}}^{j} \alpha(v_k - v_j) h^N(v_j) \right. \\
&\qquad \times \psi_{l+1}(v_1 \cdots v_{j-1}, u, v_{j+1} \cdots v_n) + \beta(u - v_j) \\
&\qquad \left. \times \prod_{\substack{k=1 \\ k\neq j}}^{n} \alpha(v_j - v_k) h'^N(v_j) \psi_{l-1}(v_1 \cdots v_{j-1}, u, v_{j+1} \cdots v_n) \right]
\end{aligned}$$

Now suppose

$$\sum e^{2\pi i l\theta} \psi_l(v_1 \cdots v_n) = \Phi_\theta(v_1 \cdots v_n)$$

Thus we get

$$\begin{aligned}
&\tau(u)\Phi_\theta(v_1 \cdots v_n) \\
&\quad = \left[e^{-2\pi i\theta} h^N(u) \prod_{k=1}^{n} \alpha(v_k - u) + e^{2\pi i\theta} h'^N(u) \prod_{k=1}^{n} \alpha(u - v_k) \right] \\
&\qquad \times \Phi_\theta(v_1 \cdots v_n) + \sum_{j=1}^{n} \left[e^{-2\pi i\theta} \left\{ \beta(u - v_j) \prod_{k=1}^{n} \alpha(v_k - v_j) h^N(v_j) \right\} \right. \\
&\qquad \left. - e^{2\pi i\theta} \beta(u - v_j) \prod_{\substack{k=1 \\ k\neq j}}^{n} \alpha(v_j - v_k) h'^N(v_k) \right] \Phi_\theta(v_1 \cdots v_{j-1}, u, v_{j+1} \cdots v_n)
\end{aligned} \tag{35}$$

Thus Φ_θ will be an eigenstate of $\tau(u)$ if and only if the second bracket in (35) vanishes; whence we get

$$\left[\frac{h(v_j)}{h'(v_j)}\right]^N = e^{4\pi i\theta} \prod_{\substack{k=1 \\ k\neq j}}^{n} \frac{n(v_j - v_k)}{\alpha(v_k - v_j)} \tag{36}$$

and the eigenvector becomes

$$\Lambda(u) = e^{-2\pi i\theta} h^N(u) \prod_{k=1}^{n} \alpha(v_k - u) + e^{2\pi i\theta} h'^N(u) \prod_{k=1}^{n} \alpha(u - v_k) \tag{37}$$

5. CONCLUSION

We have constructed an asymmetric face model corresponding to an $SU_{p,q}(2)$ invariant spin chain. The asymmetry is generated due to the parameters p and q. The intertwining relations are used to construct the Bethe eigenstates. Our analysis may give some clue to the construction of generalized face models.

ACKNOWLEDGMENT

A.G.C. is grateful to CSIR, Government of India, for a Senior Research Fellowship.

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